Hadronic contributions to $(g-2)_\mu$ from lattice QCD

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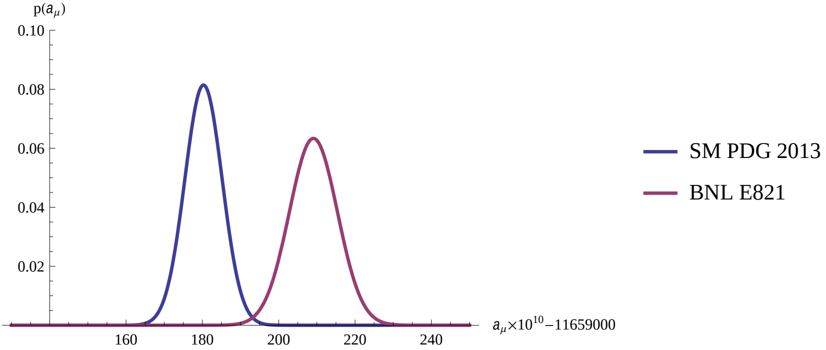
RBC and UKQCD Collaborations

February 25, 2015 – Third annual BRAIN workshop

Current status of $(g-2)_\mu$: 3.6σ tension (PDG 2013)



$2a_\mu \equiv (g-2)_\mu$


OVERSIZE LOAD

After experimental improvement?

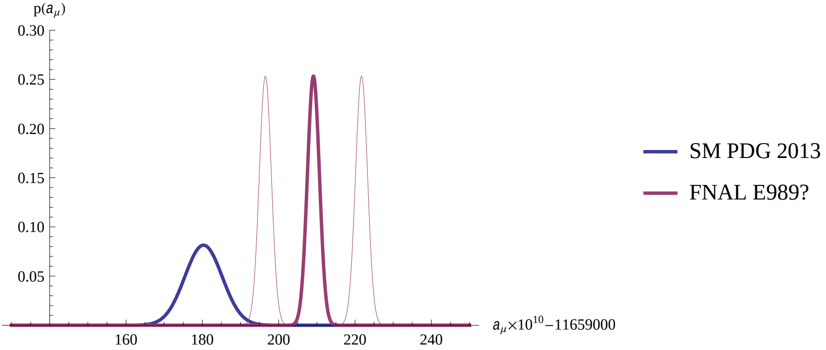
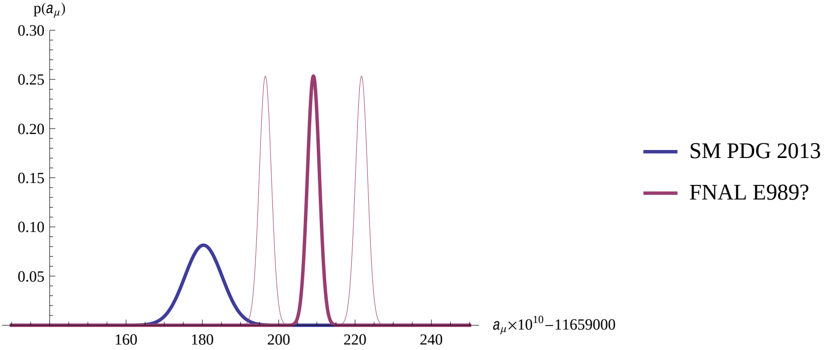


Figure shows $a_\mu^{\text{FNAL E989}}$ with $\pm 2\sigma_{\text{BNL E821}}$ variation around $a_\mu^{\text{BNL E821}}$ ($\sigma_{\text{FNAL E989}} = \sigma_{\text{BNL E821}}/4$)

After experimental improvement?



Need to solidify SM prediction and aim to match $\sigma_{\mathrm{FNAL\ E989}}$

Figure shows $a_\mu^{\mathrm{FNAL\ E989}}$ with $\pm 2\sigma_{\mathrm{BNL\ E821}}$ variation around $a_\mu^{\mathrm{BNL\ E821}}$ ($\sigma_{\mathrm{FNAL\ E989}} = \sigma_{\mathrm{BNL\ E821}}/4$)

SM prediction (PDG 2013)

Contribution	Central Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
a_μ^{QED}	11 658 471.895	0.008
a_μ^{EW}	15.4	0.1
$a_\mu^{\mathrm{HAD,\ LO\ VP}}$	* 692.3	4.2
$a_\mu^{\mathrm{HAD,\ HO\ VP}}$	-9.84	0.06
$a_\mu^{\mathrm{HAD,\ LBL}}$	** 10.5	2.6
a_μ^{SM}	11 659 180.3	4.9
FNAL E989 target		$\approx$ 1.6

* $e^+e^- \to$ hadrons (exp) and dispersion integrals; "3.3σ tension" based on: K. Hagiwara et al., J. Phys. G38 (2011) 085003: $a_\mu^{\mathrm{HAD,\ LO\ VP}} \times 10^{10} \to 694.91$

** based on Prades, de Raphael, and Vainshtein 2009 "Glasgow White Paper": QCD model including PS meson contribution; Pauk and Vanderhaeghen Eur.Phys.J. C74 (2014) 8, 3008: include AV,S,T meson poles yields $< 1.0 \times 10^{-10}$ shifts in $a_\mu^{\mathrm{HAD,\ LBL}}$

Outline

The hadronic vacuum polarization

The hadronic light-by-light contribution

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The hadronic light-by-light contribution

RBC and UKQCD collaboration on the hadronic vacuum polarization

Tom Blum (UConn)

Peter Boyle (Edinburgh)

Luigi Del Debbio (Edinburgh)

Jamie Hudspith (York)

Taku Izubuchi (BNL/RBRC)

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Christoph Lehner (BNL)

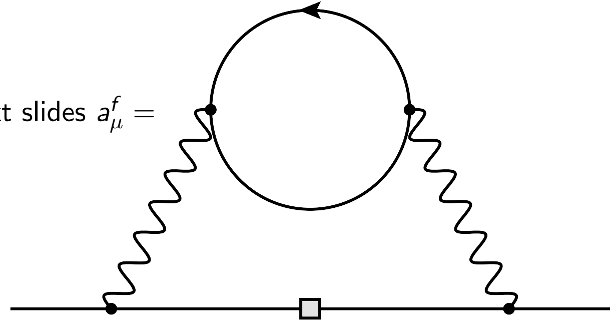
Kim Maltman (York/CSSM)

Marina Marinkovic (CERN/Southampton)

Antonin Portelli (Southampton)

The hadronic vacuum polarization (HVP) diagram

On the next slides $a_\mu^f =$



Note: there is also a quark-disconnected diagram

HPQCD 2014 (new method for improved statistics):

$a_\mu^{s/c}$	dispersion + expt	HPQCD	ETMC (prelim.)	RBC/UKQCD (prelim.)
$a_\mu^s \times 10^{10}$	55.3(8)	53.4(6)	53(3)	52.4(2.1)
$a_\mu^c \times 10^{10}$	14.4(1)	14.4(4)	14.1(6)	–

arXiv:1411.0569

	a_μ^s	a_μ^c
Uncertainty in lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
QED corrections:	0.1%	0.3%
Quark mass tuning:	0.0%	0.4%
Finite lattice volume:	$< 0.1\%$	0.0%
Padé approximants:	$< 0.1\%$	0.0%
Total:	1.1%	2.7%

Impressive progress but HVP a solved problem? No. Keep in mind total $a_\mu^{c+s+d+u} \times 10^{10} \approx 700$. Need to control light-quark case.

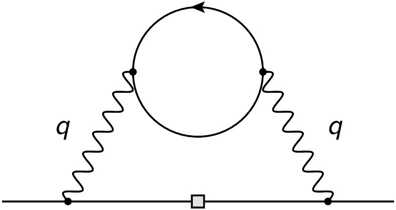
Challenges for $a_\mu^{u,d}$:

Statistics: noise problem, new methods for s, c contribution do not address main issue

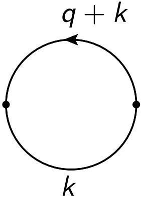
Disconnected diagram: estimated to be O(1%) of the connected contribution, see Meyer 2013

Isospin-breaking: need to include strong and EM isospin-breaking

Blum 2002

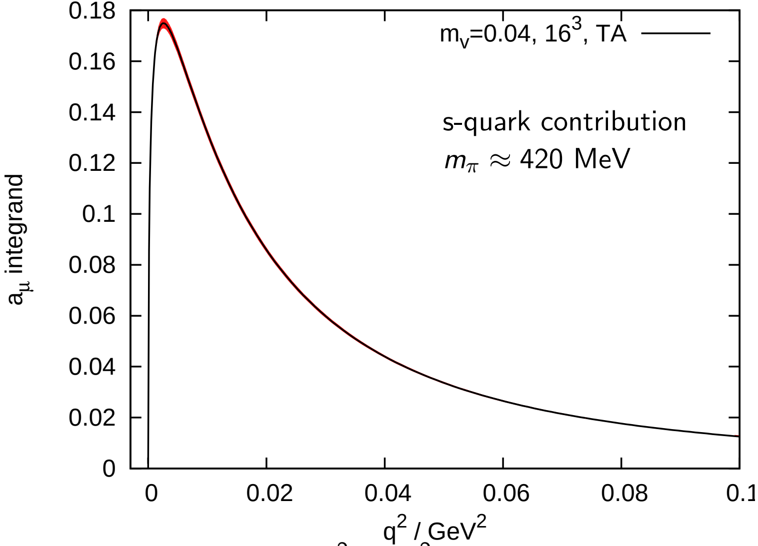


$$= \int_0^\infty d(q^2) f(q^2) \left(\frac{1}{q^2} \right.$$



$$\left. -(q \to 0) \right)$$

$$= \hat{\Pi}(q^2)$$



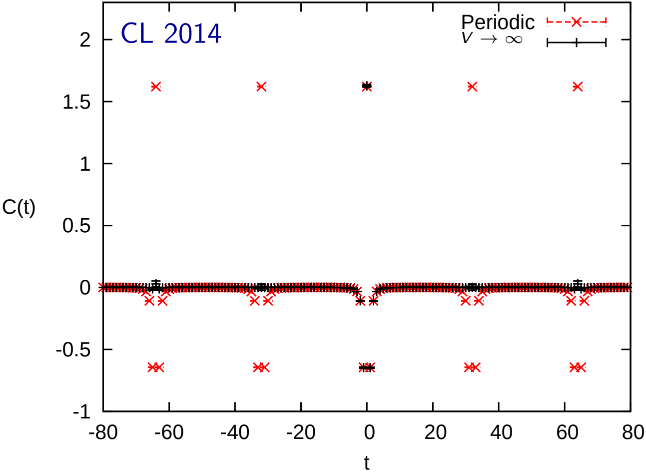
Dominant contribution from $q^2 \approx m_\mu^2$. Typically very small compared $2\pi/L$.

Source of noise for small q:

Traditional estimators do not satisfy configuration-by-configuration the properties that hold after quantum average such as $\left\langle \Pi_{\mu\nu}(q^2=0) \right\rangle = 0$, $\left\langle \mathrm{Im}\, \Pi_{\mu\nu}(q^2) \right\rangle = 0$.

$$\Pi_{\mu\nu}(x) = \left\langle V_\mu^{\text{cons.}}(x) V_\nu^{\text{loc.}}(0) \right\rangle$$

$C(t) = \sum_{\vec{x}} \Pi_{\mu\nu}(x_0 = t, \vec{x})$ for $\mu = 1, 2, 3$



Noise due to cancellation for small q^2 region ($\approx \sum_t C(t)$)

How do new methods such as the HPQCD method address the $q \to 0$ noise problem?

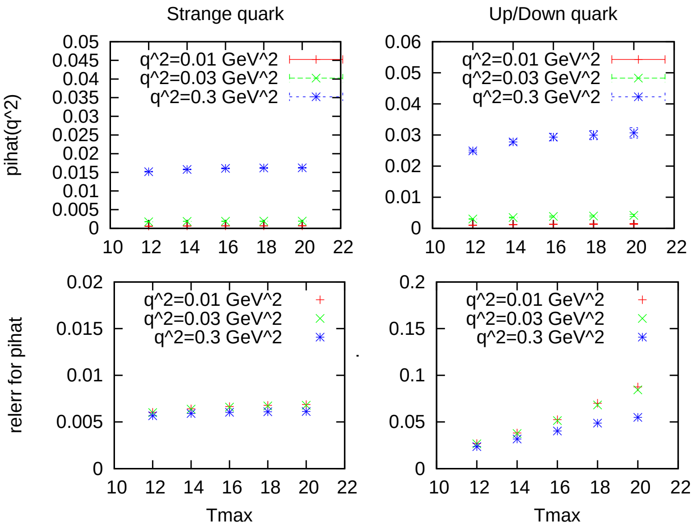
Use estimator for $\Pi_{\mu\nu}(q^2)$ and $\Pi(q^2)$ that has proper $q \to 0$ limit configuration-by-configuration:

$$\left\langle \hat{\Pi}(q^2) \right\rangle = \left\langle \sum_t \mathrm{Re}\left(\frac{exp(iqt) - 1}{q^2} + \frac{1}{2}t^2 \right) \mathrm{Re}\, C_{\mu\mu}(t) \right\rangle \qquad (1)$$

(CL lattice 2014, slight modification of Eq. (81) in Bernecker and Meyer 2012)

The HPQCD moments method is a Taylor expansion of the above estimator for small q.

But: much less important for light quarks for which different noise dominates



Data: 80 point-source measurements on RBC/UKQCD 48c physical point lattice ($a^{-1} = 1.73$ GeV); Method of Eq. (1) with $\sum_t \to \sum_{t \leq T_{\max}}$; Note: y-scale of strange/light plots!

Status of lattice HVP determinations versus precision goal

- Strange- and charm-quark contributions can be determined at experimental precision goal right now (improved estimators)
- Light quarks require much more statistics (using long-distance modeling one can carefully treat statistical for systematic errors); HPQCD/MILC, RBC/UKQCD, and others are working on this
- Still missing: disconnected diagrams (expected to be small) and isospin-breaking effects

Outline

The hadronic vacuum polarization

The hadronic light-by-light contribution

RBC and UKQCD collaboration on the hadronic light-by-light contribution

Tom Blum (UConn)

Norman Christ (Columbia)

Masashi Hayakawa (Nagoya)

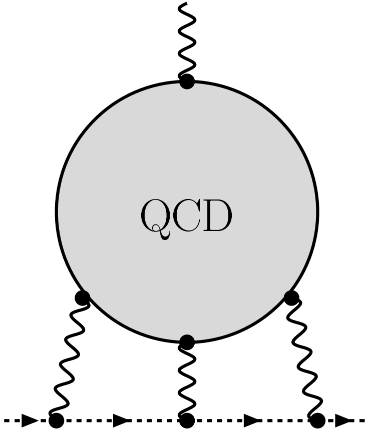
Taku Izubuchi (BNL/RBRC)

Luchang Jin (Columbia)

Christoph Lehner (BNL)

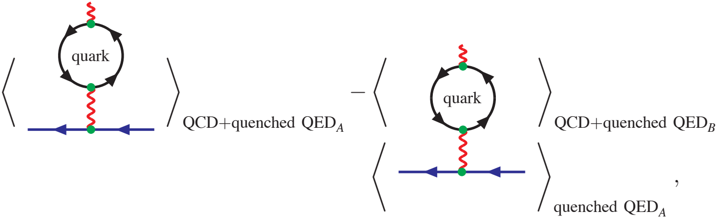
Norikazu Yamada (KEK)

The hadronic light-by-light contribution



A long-standing problem of interest for our collaboration

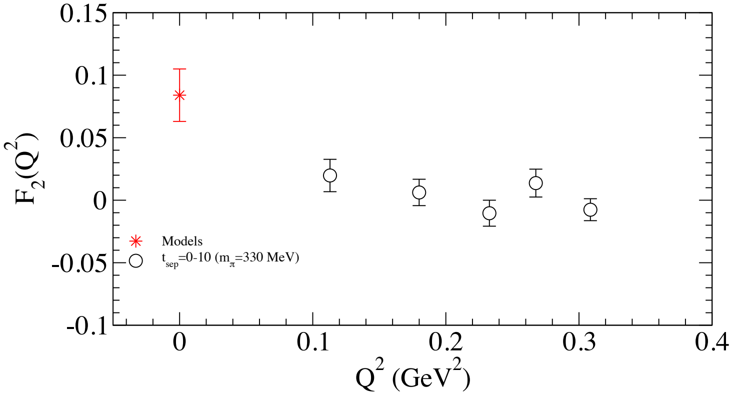
First methodology paper 10 years ago: Blum, Hayakawa, Izubuchi, Yamada: PoS(LAT2005)353 (QCD+quenched QED)



Noise control: impose quantum-average properties config-by-config ($e \to -e$, $p \to -p$)

First a-priori lattice determination:

Blum et al., Phys.Rev.Lett. 114 (2015) 1, 012001: connected diagrams only, $m_\pi = 329$ MeV, $a^{-1} = 1.73$ GeV, $L = 24^3 \times 64$



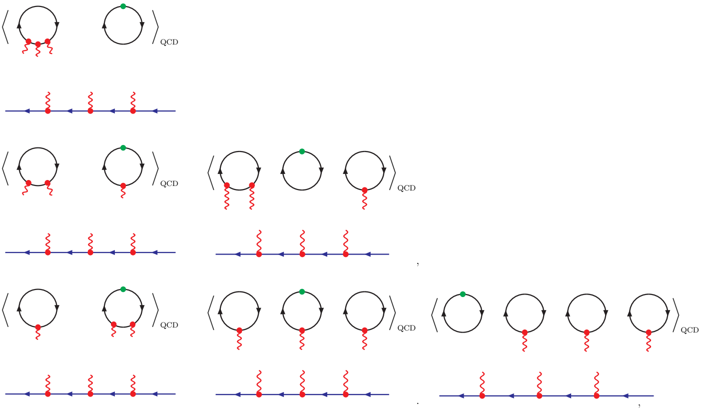
$a_\mu = F_2(0)$

Imperfections that need to be addressed:

- Omission of disconnected diagrams
- Control of large QED FV errors
- Control of excited state contributions
- Computation at physical pion mass

Inclusion of QCD+dynamical QED

Blum, Hayakawa, and Izubuchi, PoS(LATTICE 2013)439

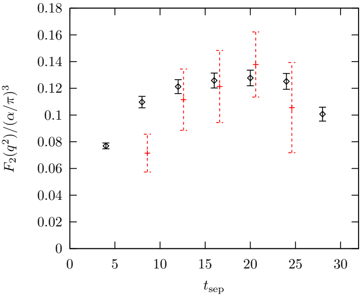


Other collaborations are generating QCD+dynamical QED ensembles: FNAL/MILC (Zhou and Gottlieb, PoS LATTICE2014 (2014) 024), BMW

Re-examine statistics

QCD+QED simulations suffer from large statistical uncertainties. We explore a different method here:

Plot for 16^3 QCD+QED data of Blum et al. 2014



Luchang Jin

Same-cost comparison: red data: old method QCD+quenched QED, black: new stochastic sampling method (Luchang Jin)

Excited states

- As we go to larger volumes, excited state contributions of $\mu + \gamma$ etc. may be enhanced
- Lattice QED perturbation theory converges well and can be used to construct improved source
- We are exploring this with the *PhySyHCAl* system that also was used for a free-field test of Blum et al. 2014

Finite-volume errors

General FV problem of QCD+QED simulations. However, for HVP computations this was no issue (see HPQCD 2014 error budget):

	a_μ^s	a_μ^c
Uncertainty in lattice spacing (w_0, r_1):	1.0%	0.6%
Uncertainty in Z_V:	0.4%	2.5%
Monte Carlo statistics:	0.1%	0.1%
$a^2 \to 0$ extrapolation:	0.1%	0.4%
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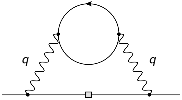
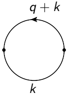
Total:	1.1%	2.7%

Benefit of treating the valence photon in infinite volume



$= \int_0^\infty d(q^2) f(q^2) \Bigg(\frac{1}{q^2}$  $-(q \to 0) \Bigg)$

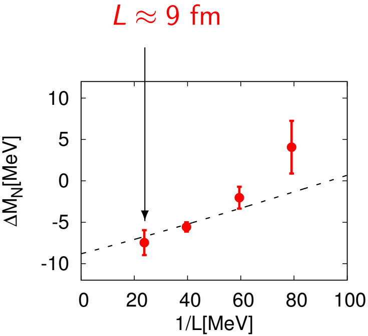
$= \hat{\Pi}(q^2)$

For LBL a similar decomposition would be much more challenging

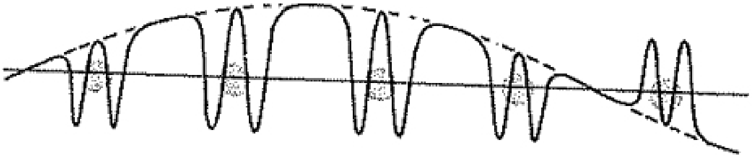
Example: dynamical QCD+QED contribution of BMW 2014

Neutron–proton mass splitting (in figure for artificially large e^2)

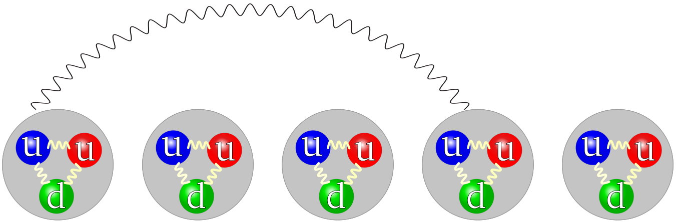
Dashed line is obtained from free fermion plus QED one-loop finite-volume pole mass shift.



Bloch's theorem and QCD+QED simulations



$\Updownarrow$



$U_\mu(x)$ $\Psi(x+\hat{L}_1+\hat{L}_2)$	$U_\mu(x)$ $\Psi(x+2\hat{L}_1+\hat{L}_2)$	$U_\mu(x)$ $\Psi(x+3\hat{L}_1+\hat{L}_2)$
$U_\mu(x)$ $\Psi(x+\hat{L}_1)$	$U_\mu(x)$ $\Psi(x+2\hat{L}_1)$	$U_\mu(x)$ $\Psi(x+3\hat{L}_1)$

Valence fermions Ψ living on a repeated gluon background U_μ with periodicity L_1, L_2 and vectors $\hat{L}_1 = (L_1, 0)$, $\hat{L}_2 = (0, L_2)$

Let ψ^θ be the quark fields of your finite-volume action with twisted-boundary conditions

$$\psi^\theta_{x+L} = e^{i\theta}\psi^\theta_x \,.$$

Then one can show that

$$\boxed{\left\langle \Psi_{x+nL}\bar{\Psi}_{y+mL}\right\rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i\theta(n-m)} \left\langle \psi^\theta_x \bar{\psi}^\theta_y \right\rangle \,, \qquad (2)}$$

where the $\langle\cdot\rangle$ denotes the fermionic contraction in a fixed background gauge field $U_\mu(x)$. (4d proof available.)

This specific prescription produces exactly the setup of the previous page, it allows for the definition of a conserved current, and allows for a prescription for flavor-diagonal states.

Status of lattice hadronic light-by-light determination:

The experimental target precision needed for the light-by-light contribution is substantially less than for the HVP contribution: $\approx 10 - 15\%$

Blum et al., Phys.Rev.Lett. 114 (2015) 1, 012001: first ab-initio computation

Work on its imperfections is in progress:

- Using the improved stochastic method to compute the connected contribution at the physical pion mass
- Exploring excited-state and finite-volume effects
- Exploring optimal strategies to include the disconnected diagrams

Other collaborations have started similar efforts (FNAL/MILC).
The lattice community is actively putting its focus on this important quantity.

Thank you